Stars and Bars Method

Stars and bars is a mathematical technique for solving certain combinatorial problems. It occurs whenever you want to count the number of ways to group identical objects.

Theorem 1 (Number of positive integer sums):

- The number of solutions to the equation X1 + X2 + … + Xk = n, such that Xi >= 1, i = {1, 2, …, n} is equal to **(n-1)C(k-1)**.

Theorem 2 (Number of non-negative integer sums):

- The number of solutions to the equation X1 + X2 + … + Xk = n, such that Xi >= 0, i = {1, 2, …, n} is equal to **(n+k-1)C(n)** or **(n+k-1)C(k-1)**.

Number of lower-bound integer sums:

- The number of solutions to the equation X1 + X2 + … + Xk = n, such that Xi >= ai, i = {1, 2, …, n} can be reduced to the following problem:

  First, we let Xi’ = Xi - ai, for i = {1, 2, …, n}.
  Now, we have the new equation X1’ + X2’ + … + Xk’ = n - a1 - a2 - … - ak,
  Xi >= 0, i = {1, 2, …, n}.

  Hence, we can find the number of solutions of the new equation using theorem 2. If we let m = n - a1 - a2 - … - ak, then we get X1’ + X2’ + … + Xn’ = m and hence, the number of solutions of the new equation is **(m+k-1)C(m)** or **(m+k-1)C(k-1)**. Therefore, the number of solutions to the original equation is **(m+k-1)C(m)** or **(m+k-1)C(k-1)**.